

Accelerated closed universes in scalar-tensor theories

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We describe an accelerating universe model in the context of a scalar-tensor theory. This model is intrinsically closed, and is filled with quintessence-like scalar field components, in addition to the Cold Dark Matter component. With a background geometry specified by the Friedman-Robertson-Walker metric, we establish conditions under which this closed cosmological model, described in a scalar-tensor theory, may look flat in a genuine Jordan-Brans-Dicke theory. Both models become indistinguishable at low enough redshift.

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I. INTRODUCTION

Recent astronomical observations conclude that the matter density related to baryonic matter and to non-baryonic cold dark matter, is much less than the critical density value [1]. From this it may be concluded that either the universe is open or that there are some other matter components which make the total matter density parameter, $\Omega_{total} \equiv \Omega_T$ (defined by the ratio between the total matter and the critical energy density), close to one. We should note that each cosmological model is defined by its content of matter, which is reflected by the value of Ω_T . On the other hand, it is well known that this parameter is related to the geometry of the universe: $\Omega_T > 1$ (closed), $\Omega_T = 1$ (flat) and $\Omega_T < 1$ (open). At the moment, we do not know precisely the amount of matter present in the universe. Therefore, still we do not know yet what the geometry of the universe is. However, combined measurements of Cosmic Microwave Background (CMB) temperature fluctuations and the distribution of galaxies on large scales strongly suggest that the universe is more likely to be flat [2, 3], consistent with the standard inflationary prediction[4].

On the other hand, recent measurements of a type Ia supernova (SNe Ia) [5, 6], at redshift $z \sim 1$, indicate that the expansion of the present universe is accelerated. This shows that in the universe there exists an important matter component that, in its most simple description, has the characteristic of a cosmological constant, i.e. a vacuum energy density which contributes to a large component of negative pressure. Other possible interpretations have been given for explaining the corresponding astronomical data. Among them, we distinguish those related to dark energy (quintessence) models, which are characterized by a scalar field χ and its potential $V(\chi)$ [7].

One characteristic of this acceleration is that it may be quite recent, since it has been determined for low redshift parameter, $0.5 < z < 1$. Beyond these redshifts the

behavior of the universe should be decelerating [8]. In this way, at different epochs in the evolution of the universe we may expect that different "matter" components dominated its evolution. Certainly, these different "matter" components should be compared not only with each other, but also with the curvature term, which defines the geometry of the universe.

In relation to the geometry of the universe, most cosmologists prefer a flat rather than a closed or an open universe. This is motivated by the mentioned redshift-distant relation for supernova of type Ia, anisotropies in the cosmic microwave background radiation [9, 10] and gravitational lensing [11], which suggest [3] that $\Omega_T = \Omega_M + \Omega_\Lambda = 1.00 \pm 0.12$ (95% cl), with Ω_M the ordinary matter density parameter, where baryons and CDM are the main contributions, and $\Omega_\Lambda \equiv \frac{\Lambda}{3H_0^2}$ is associated with a smoothly distributed vacuum energy referred to as a cosmological constant Λ , and with H_0 the actual value of the Hubble parameter (from now on the subscript zero refers to quantities evaluated at the current epoch). The expression for Ω_T given above is the main characteristic of the so called Λ CDM model.

Nothing can prevent us from thinking that this flatness might be due to a compensation among different components that enter into the dynamical equations. In fact, our main goal in this paper is to describe this idea in a Scalar-Tensor or in a version of the Jordan-Brans-Dicke (JBD) theory[12], where the JBD scalar field, ϕ has associated a scalar potential $V(\phi)$. The salient feature of this sort of theory depends on the strength of the dimensionless coupling "constant" ω that depends on the JBD scalar field ϕ in general. Here, we will consider it to be a constant that we will designate by ω_0 . At present, observational limits from the solar system measurements give $\omega_0 \gtrsim 3000$ [13]. We will fix the value of this parameter to be the quantity $\omega_0 = 3000$, throughout this paper.

We intend to start with a closed universe model com-

posed of three matter components. One of these components is the usual nonrelativistic dust matter (baryonic and CDM) ρ_M , and the other two components correspond to quintessence-type matters which we will characterize by the scalar fields Q and χ . The scalar field Q will be introduced in such a way that its dynamics will exactly cancel the curvature together with the scalar potential term associated with the JBD field, so that the resulting model will mimic a flat accelerating universe in a genuine JBD theory. Both models become indistinguishable at a low enough redshift parameter. The JBD mimicked accelerated universe will be dominated by the scalar field χ . It has the characteristics of a dark energy component, and thus, its function will be to produce the acceleration of the universe.

One of the main characteristics of the Q field is that it obeys an equation of state given by $P_Q = w_Q \rho_Q$. Here, P_Q and ρ_Q represent the pressure and the energy density, respectively. The quantity w_Q corresponds to the equation of state parameter and its range will be determined later on. Furthermore, we shall assume that this scalar field does not interact with any other field, except with the gravitational. This scalar field will appear in the fundamental field equations as a fluid component.

On the other hand, the scalar field χ , contrarily to Q , will interact not only with the gravitational field, but also with the JBD scalar field ϕ . The reason for this is that we want to obtain the correct equations of motion in the limit $\chi \rightarrow \text{const.}$, i.e. a universe model dominated by a cosmological constant in a JBD theory [14]. We will return to this point in section V.

We should mention that other authors have treated the quintessence problem in a JBD-type of theory. For example, in ref. [15] the authors took a scalar potential for a JBD field given by $V(\phi) = V_0 \phi^2$. here, they could describe an accelerating universe but without a quintessence component. In this respect, our model could be considered to be a generalization of that model since, as we will see, we will take the scalar potential to be of the form $V(\phi, \chi) = \phi V(\chi)$. As we mention above, the dependence of this potential on terms of the JBD scalar field was motivated by the fact that it gives in the limit $\chi \rightarrow \text{const.}$ a genuine cosmological constant accelerated universe described in a JBD theory. In ref. [16] the model is still more simpler in the sense that it does not consider a scalar potential for the JBD field neither quintessence, but this model presents some problems: for instance it can not describe nucleosynthesis in the radiation era. Ref. [17] apparently solves this problem. All of these models start from a flat geometry. Our model, aside from depicting a closed geometry, does not present this problem, since this model is described in such a way that for $\omega \rightarrow \infty$ and $\phi \rightarrow \text{const.}$ we achieve the results obtained in Einstein's theory of gravity [18]. The price that we must pay is the model's complexity where three scalar fields have been introduced into this model.

The plan of the paper is as follows: In section II we write down the field equations for a curved universe

model in the scalar-tensor theory. In section III we introduce the constraint equations that allow mimicking a flat universe model from a curved universe model. Section IV describes a flat accelerated universe model, in which we determine the main properties of the accelerating scalar field. Here, we also determine the deceleration parameter and the angular size as a function of time and of redshift, respectively. In section V we study a universe model dominated by a cosmological constant. Our conclusions are drawn in section VI.

II. THE JBD-TYPE FIELD EQUATIONS

We take the effective action to be given by

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[\phi R + \frac{\omega_0}{\phi} \partial_\mu \phi \partial^\mu \phi - \tilde{V}(\phi) + 16\pi \left(\frac{1}{2} \partial_\mu \chi \partial^\mu \chi - \phi V(\chi) + \mathcal{L}_{Fluid} \right) \right], \quad (1)$$

where R is the Ricci scalar curvature, $\tilde{V}(\phi)$ is the scalar potential associated with the scalar JBD field ϕ , and \mathcal{L}_{Fluid} is a classical bicomponent-fluid Lagrangian in which we include the minimally coupling scalar field Q and the CDM component. From now on we disregard any possible coupling of Q with ordinary matter, radiation, or dark matter. Notice that we have included here an interaction between the JBD field ϕ , and the dark energy field χ . The inclusion of this interaction comes motivated by the fact that we want to associate the scalar potential $V(\chi)$ with the cosmological constant Λ , in the limit $\chi \rightarrow \text{const.}$, together with the correspondence $\phi R \rightarrow \phi(R - 2\Lambda)$. Another possibility is to consider the potential associated with the JBD field $\tilde{V}(\phi)$, with a variable cosmological constant. Here we follow the former approach in which the cosmological acceleration is completely due to the scalar field χ .

The variation of the action (1) with respect to the metric tensor $g_{\mu\nu}$, the JBD field ϕ , and the Quintessence-like scalar field χ , yields the following set of Equations:

$$\begin{aligned} G_{\mu\nu} = & -\frac{8\pi}{\phi} T_{\mu\nu}^M - \frac{\omega_0}{\phi^2} \left[\partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g_{\mu\nu} \partial^\alpha \phi \partial_\alpha \phi \right] \\ & - \frac{1}{\phi} \left[D_\mu D_\nu \phi - g_{\mu\nu} \square \phi + \frac{1}{2} g_{\mu\nu} \tilde{V}(\phi) \right] \\ & - \frac{8\pi}{\phi} \left[\partial_\mu \chi \partial_\nu \chi - \frac{1}{2} g_{\mu\nu} \partial^\alpha \chi \partial_\alpha \chi + g_{\mu\nu} \phi V(\chi) \right], \\ & \square \phi - \frac{1}{2\phi} \partial_\alpha \phi \partial^\alpha \phi + \frac{\phi}{2\omega_0} \frac{\partial \tilde{V}(\phi)}{\partial \phi} \end{aligned} \quad (2)$$

$$= -\frac{\phi}{2\omega_0} [16\pi V(\chi) - R], \quad (3)$$

$$\square\chi + \phi \frac{\partial V(\chi)}{\partial \chi} = 0 \quad (4)$$

and, for each fluid component we have a conservation law

$$T_{\mu\nu}^{i;\nu} = 0 \quad (i = 1, 2). \quad (5)$$

In these Equations, $G_{\mu\nu}$ is the Einstein tensor, R the scalar curvature, $T_{\mu\nu}^M$ the matter stress tensor associated with the two fluid lagrangian, \mathcal{L}_{Fluid} , and $\square \equiv D_\alpha D^\alpha = \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\alpha} (\sqrt{-g} g^{\alpha\beta} \partial_\beta)$.

If we assume that the spacetime is isotropic and homogeneous with metric corresponding to the standard Friedman-Robertson-Walker (FRW) metric

$$ds^2 = dt^2 - a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2\theta d\varphi^2) \right], \quad (6)$$

where $a(t)$ represents the scale factor and the parameter k takes the values $k = -1, 0, 1$ corresponding to an open, flat, and closed three-geometry, respectively, and considering also that the JBD field ϕ is homogeneous, i.e. is a time-depending quantity only, (the same is assumed for the other fields) the set of Equations (2) - (4) yields the following field Equations:

$$3 \left(\frac{\dot{a}}{a} \right)^2 + 3 \frac{k}{a^2} = \frac{1}{2\phi} \left[\frac{\omega_0}{\phi} (\dot{\phi})^2 + \tilde{V}(\phi) - 6 \frac{\dot{a}}{a} \dot{\phi} + 16\pi \left(\rho_{Fluid} + \frac{1}{2} \dot{\chi}^2 + \phi V(\chi) \right) \right], \quad (7)$$

$$2 \frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2} = -\frac{1}{2\phi} \left[\frac{\omega_0}{\phi} (\dot{\phi})^2 + 2\ddot{\phi} + 4 \frac{\dot{a}}{a} \dot{\phi} - \tilde{V}(\phi) + 16\pi \left(p_{Fluid} + \frac{1}{2} \dot{\chi}^2 - \phi V(\chi) \right) \right], \quad (8)$$

$$\ddot{\phi} + 3 \frac{\dot{a}}{a} \dot{\phi} + \frac{1}{3 + 2\omega_0} \left[\frac{\partial \tilde{V}(\phi)}{\partial \phi} - \frac{2\tilde{V}(\phi)}{\phi} \right]$$

$$= \frac{8\pi}{3 + 2\omega_0} \frac{1}{\phi} [\rho_{Fluid} - 3p_{Fluid} - \dot{\chi}^2 + 2\phi V(\chi)], \quad (9)$$

and

$$\ddot{\chi} + 3 \frac{\dot{a}}{a} \dot{\chi} = -\phi \frac{\partial V(\chi)}{\partial \chi}. \quad (10)$$

From Eq. (5), the continuity equations for each individual fluid component are given by

$$\dot{\rho}_i = -3 \frac{\dot{a}}{a} (\rho_i + P_i) \quad (i = 1, 2). \quad (11)$$

These fluid components represent, on the one hand, the usual (Cold Dark) matter component (ρ_M, P_M) with equation of state $P_M = 0$ (dust), and therefore $\rho_M(t) \propto a^{-3}(t)$. On the other hand, we have the quintessence-like scalar field $Q(t)$. For this latter field we define a density and pressure by

$$\rho_Q = \frac{1}{2} \dot{Q}^2 + V(Q), \quad (12)$$

and by

$$P_Q = \frac{1}{2} \dot{Q}^2 - V(Q), \quad (13)$$

respectively. This scalar field obeys an equation of state given by $P_Q = w_Q \rho_Q$. In the following, we consider this parameter to be constant, and we will determine its range of values in the next section. Here also, the gravitational "constant" becomes given by

$$G(t) = 2 \left(\frac{\omega_0 + 2}{2\omega_0 + 3} \right) \frac{1}{\phi(t)}. \quad (14)$$

This latter expression fixes the present value of the JBD scalar field ϕ in terms of the Newton constant G and the JBD parameter ω_0 .

III. THE FLATNESS CONSTRAINT EQUATIONS

As was specified in the introduction, we want to describe a curved universe which, at low redshift, mimicks a flat universe model. In order to do this, we substitute the fluid components into the field Equations, and we extract the following "flatness constraint Equations":

$$3 \frac{k}{a^2} = \frac{8\pi}{\phi} \rho_Q + \frac{\tilde{V}(\phi)}{2\phi}, \quad (15)$$

$$-\frac{k}{a^2} = \frac{8\pi}{\phi} P_Q - \frac{\tilde{V}(\phi)}{2\phi}, \quad (16)$$

and

$$\frac{8\pi}{\phi} (\rho_Q - 3P_Q) = \frac{\partial \tilde{V}(\phi)}{\partial \phi} - 2 \frac{\tilde{V}(\phi)}{\phi}. \quad (17)$$

With these conditions the following set of dynamical field equations occurs:

$$3 \left(\frac{\dot{a}}{a} \right)^2 - 8\pi V(\chi) = \frac{8\pi}{\phi} \left[\rho_M + \frac{1}{2} \dot{\chi}^2 \right] + \frac{\omega_0}{2} \left(\frac{\dot{\phi}}{\phi} \right)^2 - 3 \frac{\dot{a}}{a} \frac{\dot{\phi}}{\phi}, \quad (18)$$

$$-2 \frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a} \right)^2 + 8\pi V(\chi) = \frac{4\pi}{\phi} \dot{\chi}^2 + \frac{\omega_0}{2} \left(\frac{\dot{\phi}}{\phi} \right)^2 + \frac{\ddot{\phi}}{\phi} + 2 \frac{\dot{a}}{a} \frac{\dot{\phi}}{\phi}, \quad (19)$$

and

$$\frac{\ddot{\phi}}{\phi} + 3 \frac{\dot{a}}{a} \frac{\dot{\phi}}{\phi} = \frac{8\pi}{3+2\omega_0} \left[\frac{1}{\phi} (\rho_M - \dot{\chi}^2) + 2V(\chi) \right]. \quad (20)$$

To this set of Equations we should add Eq. (10) which does not change. Therefore, our fundamental set of Equations is formed by Eqs. (18) - (20) together with Eq. (10). We should stress here that this set of Equations correspond to a genuine JBD theory. Therefore, we have passed from a curvature (closed) model, described by using a scalar-tensor theory (characterized by a scalar potential), to a flat model, described by using the JBD theory.

At this point we might specify that the constraint Eqs (15) - (17) may be looked upon as assumptions meant for some simplification of the Equations of motions described by Eqs (7) - (9).

Before studying this set of Eqs. of motion we want to describe the characteristics of either of the scalar fields Q and ϕ by using the constraint Eqs. (15) - (17).

First of all, adding Eqs. (15) and (16) and using the Equation of state $P_Q = w_Q \rho_Q$, we obtain an expression that, when evaluated at present time, we get

$$\Omega_Q = \frac{4}{3\kappa} \left(\frac{\Omega_k}{1+w_Q} \right), \quad (21)$$

where κ , Ω_Q and Ω_k become defined by $\kappa = \frac{3+2\omega_0}{2+\omega_0}$, $\Omega_Q = \left(\frac{8\pi G}{3H_0^2} \right) \rho_Q^0$ and $\Omega_k = \frac{k}{H_0^2 a_0^2}$, respectively. Now, since $k \neq 0$ and $|w_Q| < 1$, we should take $k = 1$, otherwise the energy density ρ_Q would be negative, violating the strong energy condition. Therefore, in the following we will restrict ourselves to closed universe models. On the other hand, by subtracting Eqs. (15) and (16) and using again the Equation of state for the Q field, we get

$$\rho_Q = \frac{1}{8\pi} \left(\frac{1}{\sigma} \right) \tilde{V}(\phi), \quad (22)$$

where σ becomes defined by $\sigma = 1 + 3w_Q$. This latter expression tells us that we must have $w_Q > -1/3$ in order to make ρ_Q a positive quantity. Thus, we can say that the range of the w_Q parameter corresponds to $-1/3 < w_Q \leq 1$.

Combining Eqs. (17) and (22), we can obtain an explicit expression for the JBD scalar potential:

$$\tilde{V}(\phi) = \tilde{V}_0 \left(\frac{\phi}{\phi_0} \right)^{3 \left(\frac{1+w_Q}{\sigma} \right)}. \quad (23)$$

Here, the constant of integration has been fixed by asking that the scalar potential at present time becomes \tilde{V}_0 .

With the help of this latter relation we could get a more precise value of the constant w_Q . By using Eq. (23) together with the constraint Eqs. (15) and (16) we get a relation between the scale factor a and the JBD field ϕ given by $\left(\frac{1+w_Q}{\sigma} \right) a^2 = \frac{\phi_0}{\tilde{V}_0} \left(\frac{\phi}{\phi_0} \right)^{-2/\sigma}$, which yields the following relation:

$$H(t) = \left(\frac{1}{\sigma} \right) \epsilon(t), \quad (24)$$

where $H(t) = \dot{a}/a$ is the hubble parameter and $\epsilon(t) = -\dot{\phi}/\phi = G(t)/G(t)$ represents the changing rate of the gravitational constant. When this expression is evaluated at present time, we get for the parameter w_Q the following quantity:

$$w_Q = -\frac{1}{3} + \frac{1}{3} \frac{\epsilon_0}{H_0}. \quad (25)$$

Therefore, this parameter becomes determined by the ratio of the present time variation of the gravitational constant and the Hubble parameter. Local laboratory and solar system experiments put an upper limit on the ϵ_0 parameter given by $\epsilon_0 \leq \pm 10^{-11}$ per year [19]. This limit together with the value measured for the Hubble parameter, which is in the range $56 \text{ [Km/s/Mpc]} \leq H_0 \leq 88 \text{ [Km/s/Mpc]}$ according to the 2σ range of the HST Key Project [20], induce a limit for the w_Q parameter. We will consider a positive value for ϵ_0 (the gravitational "constant" is an increasing function of time). In order to agree with the bound specified for w_Q previously, we will use the value $w_Q = -0.3324$, which corresponds to $\epsilon_0 = 10^{-13}$ per year and $H_0 = 72 \text{ [Km/s/Mpc]}$.

Since our interest is to describe an accelerating universe, we will assume that a is a function of the cosmic time t in the form $a(t) = a_0 \left(\frac{t}{t_0} \right)^N$, with $N \gtrsim 1$. With this assumption, together with the constraint Eqs. and the Equation of state for the scalar field Q , we find that $Q(t) = \bar{Q} \left[1 - \left(\frac{t_0}{t} \right)^{\alpha-1} \right] + Q_0$ and $V_Q(t) = V_Q^0 \left(\frac{t_0}{t} \right)^{2\alpha}$, where Q_0 is the present value of Q , $\alpha = \frac{3}{2} N(1+w_Q)$, $\bar{Q} = \left(\frac{\alpha-1}{t_0} \right) \sqrt{\frac{8\pi}{\tilde{V}_0} \left(\frac{\sigma}{1+w_Q} \right)}$, and $V_Q^0 = \frac{\tilde{V}_0}{16\pi} \left(\frac{1-w_Q}{\sigma} \right)$.

These relations allow us to write an explicit expression for the scalar potential associated with Q :

$$V(Q) = V_Q^0 [1 - \bar{Q}(Q - Q_0)]^{\frac{2\alpha}{\alpha-1}}. \quad (26)$$

Figure (1) shows the form of the potential for four different values of N . We note that the potential decreases when Q increases, tending to a vanishing value for $Q \rightarrow \infty$. Note also that as we increase the value of N , $V(Q)$ tends to zero faster. Asymptotically, this scalar field behaves as a stiff fluid ($P_Q = \rho_Q$), for $\dot{Q} \neq 0$.

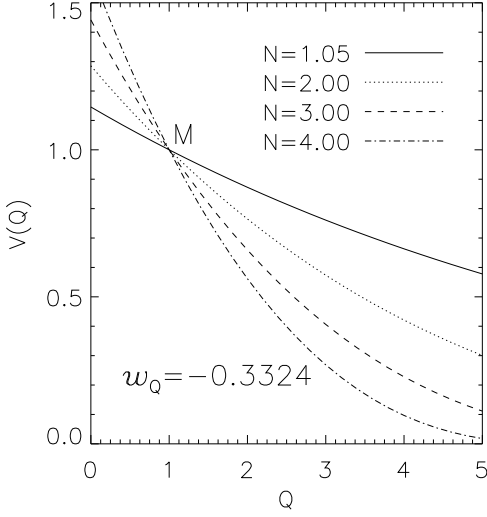


FIG. 1: Plot of the scalar potential V_Q (in units of $\frac{\tilde{V}_0}{16\pi} \left(\frac{1-w_Q}{\sigma} \right)$) as a function of the scalar field Q (in unit of Q_0), for four different values of the parameter N . We have taken $w_Q = -0.3324$. The point M , where all the curves intercept, coincides with $Q_0 \equiv t_0 \sqrt{\tilde{V}_0/8\pi}$.

IV. AN ACCELERATED UNIVERSE MODEL

Starting from the field Eqs. (18) - (20), together with Eq. (10), we want to describe the main characteristics of our accelerated model. Specifically, we want to determine the characteristics of the scalar field χ . By using the accelerated power law solution, it is not hard to find that the scalar field χ and its associated potential $V(\chi)$ become given by

$$\chi(x) = \sqrt{\frac{1}{\pi G}} \frac{\eta_1}{2\beta} x^{-\beta} \times {}_2F_1 \left(\left[-\frac{\beta}{\gamma} - \frac{1}{2} \right], \left[\frac{\gamma - \beta}{\gamma} \right], A_1 x^\gamma \right) \quad (27)$$

and

$$V(x) = \left(\frac{H_0^2}{16\pi} \right) \eta_2 \left[x^{-2/N} - A_1 x^{-\delta} \right], \quad (28)$$

respectively. Here, the constants are given by $\beta = 3(\frac{1}{2} + w_Q)$, $\gamma = -2(1 - 1/N) + 3w_Q$, $\delta = 2 - 3w_Q$, $\eta_1 = \sqrt{\eta_2 \kappa (1 + w_Q)}$, $\eta_2 = 3 - \frac{w_0}{2} \sigma^2 - 3\sigma$ and $A_1 = \frac{3}{2} \kappa \frac{\Omega_M}{\eta_2}$. The minimal value of the scalar field χ , where the potential $V(\chi)$ vanishes, corresponds to the value

$$\chi_{Min} = \left\{ \frac{3\kappa}{2} \left(\frac{\Omega_M}{3 - \frac{w_0}{2} \sigma^2 - 3\sigma} \right) \right\}^{1/(3-\sigma-2/N)}.$$

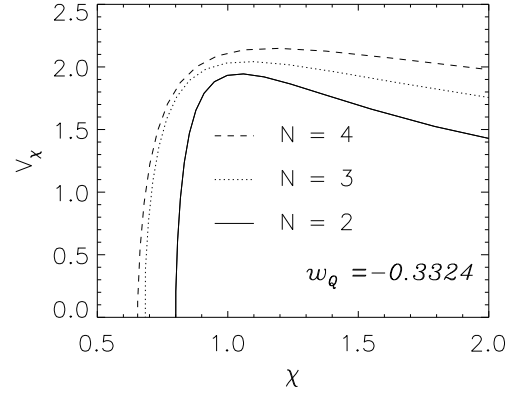


FIG. 2: This graph shows the scalar potential V_χ (in units of $(16\pi t_0^2)^{-1}$) as a function of the scalar field χ (in units of $(2\pi G_N)^{-1/2}$) for different values of the parameter N , as is shown in the figure. Here, we have taken $\Omega_M = 0.35$, $w_Q = -0.3324$ and $w_0 = 3000$.

By using numerical computations we can plot the scalar potential $V_Q \chi$ as a function of the scalar field $Q \chi$. Figure (2) shows the potential $V(\chi)$ as function of the scalar field χ for four different values of the parameter N , and the other parameters w_0 and w_Q have been fixed at 3000 and -0.3324, respectively. All of these curves asymptotically tend to vanish for $\chi \rightarrow \infty$. There, in this limit and for $\dot{\chi} \neq 0$, the universe becomes dominated by a stiff fluid with Equation of state $P_\chi = \rho_\chi$. This property also is found in similar models described by using Einstein's theory of gravity [18].

If we associate with the scalar field χ an energy and pressure density defined by $\rho_\chi = \frac{1}{2} \dot{\chi}^2 + \phi V(\chi)$ and $P_\chi = \frac{1}{2} \dot{\chi}^2 - \phi V(\chi)$, respectively, we can introduce an equation of state parameter w_χ defined by the ratio P_χ/ρ_χ . It is not hard to see that this quantity is negative for small redshifts. This means that the scalar field χ acts as the source for the acceleration of the universe. In general this quantity is variable and becomes a constant only for $N = \frac{2}{2-3w_Q}$. In any case its present value

will be interesting at the time of calculating the present deceleration parameter q_0 , as we will see soon.

From the definition of the deceleration parameter $q(t) = -\frac{\ddot{a}(t)}{a(t)H(t)^2}$ together with the field Equation of motion, Eqs. (19), we obtain

$$q(t) = \frac{1}{2} + \frac{4\pi}{3+2w_0} \frac{\rho_M}{H^2\phi} + 4\pi w_x \left(\frac{1+2w_0}{3+2w_0} \right) \frac{\rho_x}{H^2\phi} + \frac{w_0}{4} \left(\frac{\epsilon}{H} \right)^2 + \frac{1}{2} \left(\frac{\epsilon}{H} \right). \quad (29)$$

This expression, together with Eq. (18), when evaluated at present time, gives

$$q_0 = \frac{1}{2} \left(\frac{3+w_0}{2+w_0} \right) \Omega_M + \frac{1}{4} \kappa \Omega_\chi \left[1 + 3 \left(\frac{1+2w_0}{3+2w_0} \right) w_x^0 \right] + \frac{w_0}{3} \left(\frac{\epsilon}{H} \right)_0^2 + \left(\frac{\epsilon}{H} \right)_0, \quad (30)$$

where w_x^0 represents the present equation of state parameter. Note that we have introduced the present density parameter Ω_χ defined by $\Omega_\chi = \frac{8\pi G}{3H_0^2} \rho_x^0$. In order to describe an acceleration for our model, we need to satisfy for the w_x^0 parameter the following inequality:

$$w_x^0 < -\frac{1}{3} \left(\frac{3+2w_0}{1+2w_0} \right) \left[1 + 2 \left(\frac{3+w_0}{3+2w_0} \right) \frac{\Omega_M}{\Omega_\chi} \right],$$

which coincides with the result found in Einstein's theory of gravity, in the limit $w_0 \rightarrow \infty$. Note that here we have neglected the ratio $\left(\frac{\epsilon}{H} \right)_0$, (and its square).

Now we would like to calculate the angular size $\Theta(z)$ as a function of the redshift z . In order to do this, we need first to calculate the luminosity distance $d_L(z)$. This parameter plays a crucial role in describing the geometry and matter content of the universe. From the metric (6), we observe that light emitted by the object of luminosity \mathcal{L} and located at the coordinate distance θ at a time t , is received by an observer (assumed located at $\theta = 0$) at the time $t = t_0$. The time coordinates are related by the cosmological redshift z in the θ direction by the expression: $1+z = a_0/a(t)$. The luminosity flux reaching the observer is $\mathcal{F} = \mathcal{L}/4\pi d_L^2$, where d_L is the luminosity distance to the object, given by $d_L(z) = a_0 \sin[\theta(z)](1+z)$. On the other hand, if we want to obtain an explicit expression for the angular size, let us now consider an object aligned to the φ direction and proper length l , so that its "up" and "down" coordinates are $(\theta, \varphi + \delta\varphi, 0)$ and $(\theta, \varphi, 0)$. The proper length of the object is obtained by setting $t = \text{const.}$ in the line-element metric (6), $ds^2 = -l^2 = -a^2(t) \sin^2(\theta) \delta\varphi^2$. Thus, the angular size becomes $\delta\varphi \equiv \Theta(z) = \frac{l}{d_L(z)} (1+z)$. Our solution gives

$$\Theta(z) = l H_0 \frac{\bar{\alpha}(1+z)}{\sin \left\{ \bar{\alpha} \left[(1+z)^{\bar{N}} - 1 \right] \right\}}, \quad (31)$$

where $\bar{N} = 1 - 1/N$ and $\bar{\alpha} = \frac{3}{4}(1+w_Q)\kappa\Omega_Q$.

Figure 3 shows the angular size as a function of the redshift for three different values of the parameters N ($N=2, 3, 4$). We have used the value $\Omega_Q = 0.02$. For comparison, in this plot we have added the graph of the angular size corresponding to the flat FRW model for $N=4$. Notice that, at sufficient large redshift, the two curves begin to separate. Therefore, we expect that we could distinguish them at high enough redshift.

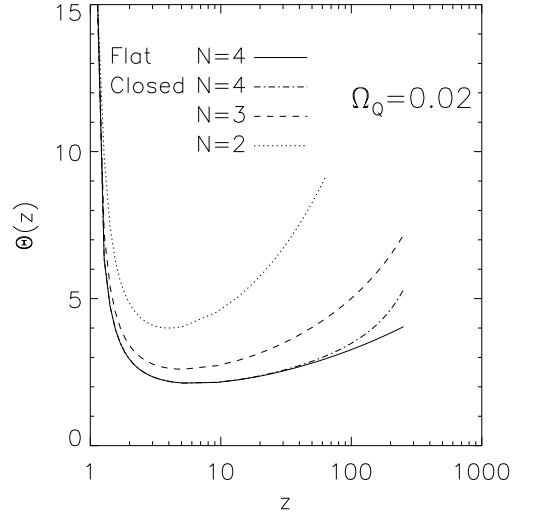


FIG. 3: This plot shows the angular size $\Theta(z)$ (in units of lH_0) as a function of the redshift z , for different values of the parameter N ($N=2,3$ and 4). We have included in this plot the angular size for a flat model in the case $N = 4$. Notice that at low enough redshift the models become indistinguishable.

V. COSMOLOGICAL CONSTANT UNIVERSE MODEL

Here we study the case in which $\chi = \chi_0 = \text{const.}$ and taking $8\pi V(\chi_0) = \lambda$. The set of Equations (18) - (20) reduces to

$$3 \left(\frac{\dot{a}}{a} \right)^2 - \lambda = \frac{8\pi}{\phi} \rho_M + \frac{\omega_0}{2} \left(\frac{\dot{\phi}}{\phi} \right)^2 - 3 \frac{\dot{a}}{a} \frac{\dot{\phi}}{\phi} \quad (32)$$

$$-2 \frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a} \right)^2 + \lambda = \frac{\omega_0}{2} \left(\frac{\dot{\phi}}{\phi} \right)^2 + \frac{\ddot{\phi}}{\phi} + 2 \frac{\dot{a}}{a} \frac{\dot{\phi}}{\phi}, \quad (33)$$

and

$$\frac{\ddot{\phi}}{\phi} + 3 \left(\frac{\dot{a}}{a} \right) \left(\frac{\dot{\phi}}{\phi} \right) = \frac{2\lambda}{3+2\omega_0} + \frac{8\pi}{\phi} \frac{\rho_M}{3+2\omega_0}. \quad (34)$$

This set of Eqs. coincides with that studied in Ref. [14]. There, dust ($P_M \approx 0$) was considered for the matter component. The case $\Lambda = 0$ was treated in Ref [21].

In order to write down a possible solution for the set of Equations (32) - (34) we first notice that the quantity

$$B_1 = \sqrt{1 - \frac{2}{4 + 3w_0} \left(\frac{2}{\kappa}\right) \left(\frac{\Omega_\chi}{\Omega_M^2}\right) [1 + (1 + w_0)\sigma]^2},$$

with Ω_χ , Ω_M , κ and σ defined in the previous section, is real for the present values of the quantities that we are considering. Therefore, the solution to this set of Equations which vanishes at $t = 0$ becomes [14]

$$a(t) = a_0 \left\{ \frac{B_1 \cosh[2\kappa_\chi(x - x_c)] - 1}{B_1 \cosh[2\kappa_\chi(x_0 - x_c)] - 1} \right\}^{\alpha_1} \times \left\{ \frac{\frac{B_2 \tanh[\kappa_\chi(x - x_c)] - 1}{B_2 \tanh[\kappa_\chi(x - x_c)] + 1}}{\frac{B_2 \tanh[\kappa_\chi(x_0 - x_c)] - 1}{B_2 \tanh[\kappa_\chi(x_0 - x_c)] + 1}} \right\}^{\alpha_2}, \quad (35)$$

where $x = H_0 t$, $\kappa_\chi = \frac{1}{2}\bar{\eta}\sqrt{3\Omega_\chi}$, $B_2 = \sqrt{\frac{1+B_1}{1-B_1}}$, $x_c = -\frac{1}{\kappa_\chi} \tanh^{-1}(B_2^{-1})$, $\alpha_1 = \frac{1+w_0}{4+3w_0}$, $\alpha_2 = \frac{1}{4+3w_0} \sqrt{\frac{3+2w_0}{2}}$ and $\bar{\eta} = \sqrt{2 \left(\frac{4+3w_0}{3+2w_0} \right)}$.

From this expression we can determine the deceleration parameter $q(t)$ as we defined it previously. The following Figure shows how this parameter changes with time for three different values of the density parameter $\Omega_\chi = \frac{16\pi V_\chi^0}{3H_0^2}$.

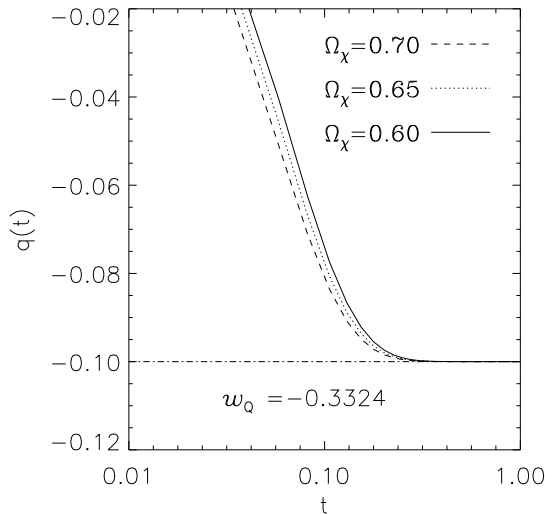


FIG. 4: This plot shows the deceleration parameter $q(t)$ as a function of time t (in unit of H_0), for three different values of the parameter Ω_χ (0.60, 0.65, 0.70).

We should note that, when a value for the parameter

Ω_χ is given, the parameter Ω_M becomes immediately determined, since this parameter has to satisfy the relation

$$\Omega_M = \frac{2}{\kappa} \left(1 - \sigma \left[1 + \frac{w_0}{6} \sigma \right] \right) - \Omega_\chi.$$

This relation results from the field Eq. (32) when it is evaluated at the present time. Note also that this relation reduces to the expression $\Omega_M + \Omega_\chi = 1$, in the limit of Einstein's theory.

Note also that the deceleration parameter tends to the value -0.1 at the current epoch, as shown from Fig. 4, implying that the expansion of the universe is accelerating rather than slowing down. This certainly agrees with evidence coming from type Ia supernova observational data [5, 6]. Finally, we should mention that, in the limit of $\phi \rightarrow \text{const.}$ and $\omega \rightarrow \infty$, the solution of Eq. 35 becomes

$$a(t) \sim \sinh^{2/3} \left(\frac{3}{2} \sqrt{\Omega_\chi} H_0 t \right), \quad (36)$$

which is nothing but the exact solution to the Friedmann Equations [14].

VI. CONCLUSIONS

We have described a closed universe model in which, apart from the usual Cold Dark Matter component, we have included a quintessence-like scalar field Q in a scalar-tensor theory characterized by a scalar JBD field ϕ and its scalar potential $V(\phi)$. We have fine-tuned the Q component, together with the curvature term and the scalar potential $V(\phi)$, for mimicking a flat universe model. The resulting model corresponds to the Quintessence (or Dark Energy) Cold Dark Matter (χ CDM) scenario described in a JBD theory of gravity, in which the JBD parameter w_0 has the value $w_0 = 3000$, in agreement with solar experiments. The flatness conditions allow us to obtain the properties of the scalar field Q together with the JBD scalar field ϕ . Especially, we have determined an explicit form for the potentials $V(Q)$ and $V(\phi)$. There, we have imposed an effective equation of state, $P_Q = w_Q \rho_Q$, for the field Q , in which the equation of state parameter w_Q has taken the value $w_Q = -0.3324$. The main characteristic of $V(Q)$ is that it decreases when Q increases, going to zero asymptotically. Contrarily, the potential $V(\phi)$ occurred as an increasing quantity when ϕ increases.

After applying the flatness constraint Equations for our model, we determined the angular size, apart from the deceleration parameter. There it was shown that, at low enough redshifts, the curvature and the flat models become indistinguishable and, on the other hand, that the model presents an acceleration rather than a deceleration. The same parameter of deceleration was determined in the limit in which the scalar field $\chi = \text{const} = \chi_0$, in which the potential term contributed as a cosmological constant, in the identification $16\pi V(\chi_0) = \lambda$.

The resulting Equations of motion coincided with those studied by Uehara and Kim. With the values used for the present quantities, we have found that our model perfectly accommodates an accelerating universe model in a genuine JBD theory.

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